<Chapter 4>

Expanding the model - From Competition to Cooperation

Up until this point, we have seen how positive competition coefficients have an effect of limiting the growth of competitors. But what happens if the competition coefficient is a negative number? This is discussed in this chapter.

$$\left.\begin{aligned} dx/dt &= r_1x(1-(x+by)/K_1) \\ dy/dt &= r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

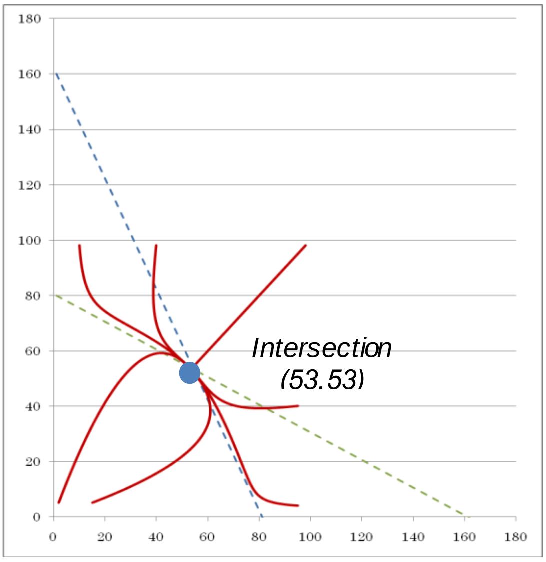
As we learned in Chapter 2, ***a*** is the degree of effect that ***x*** has on ***y***'s growth, and ***b*** is the degree of effect that ***y*** has on ***x***'s growth. If both ***a*** and ***b*** are negative numbers, the numerator will be relatively smaller than that of positive numbers against the denominator ***K***. As the numerator gets smaller, the results of the calculation $(1-(x+by)/K_1)$ and $(1-(ax+y)/K_2)$ will get larger, and so the rate of change *dx/dt, dy/dt* will increase. In other words, the growth of ***x*** and ***y*** will be assisted by each other's presence. In biology, this is called *"symbiosis".* In the business world, there are many examples of this, such as outsourcing, which provides reciprocal benefits to both the outsourcer and its contractor. The reason that industry groups, shopping arcades and small stores still exist is naturally because businesses of the same industry coming together creates more stability for the industry as a whole. It is easy to understand that instead of avoiding competitors and opening a store in a quiet alleyway, it is more profitable to open a store in a busy shopping area, despite there being competitors. On the other hand, buying out competitors through M&A can result in competition itself going away all together. In other words, M&A can also be thought of as a strategy to switch a positive competition coefficient to a negative one.

Let's see the effects in a calculation. If we use the same combination of initial values that was used in Chapter 2 and change the competition (cooperation) coefficient from *0.5* to ***-0.5*** *(Table 13)*, the results will be as shown in *Figure 25*. *Figure 24-1* in *Chapter 2* looks a bit like *a spider* with seven legs, but *Figure 24-2* looks like someone has pinned down the legs and moved the point of intersection towards the top right like *a witch's broom*. The point of coexistence (intersection coordinates) was *(53,53)* when the competition coefficient was *positive 0.5* for both companies, but by making the competition coefficient *negative -0.5*, the point of coexistence becomes three times higher *(160,160)*. In other words, if two species are competing for survival, the total number of organisms will be *106*, whereas it increases to *320* when the species are cooperating.

Table 13

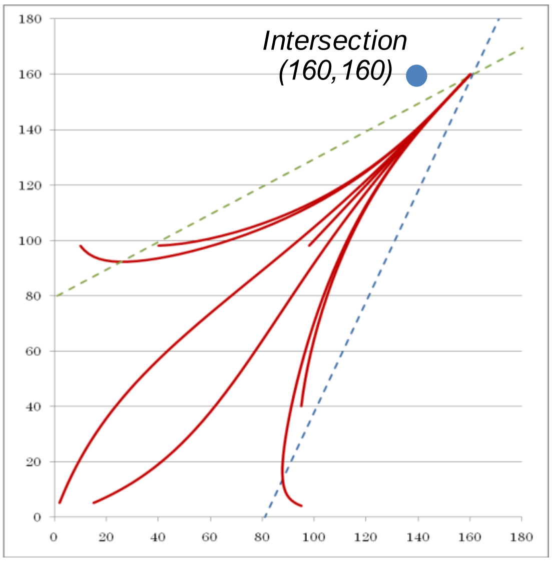
	Growth Coefficient		*Environmental Capacity*		*Competition Coefficient*	
x	r_1	*0.5*	K_1	*80*	*a*	**0.5 → -0.5**
y	r_2	*0.5*	K_2	*80*	*b*	**0.5 → -0.5**

Figure 24-1



r1,r2=0.5, K1,K2=80, a,b=0.5

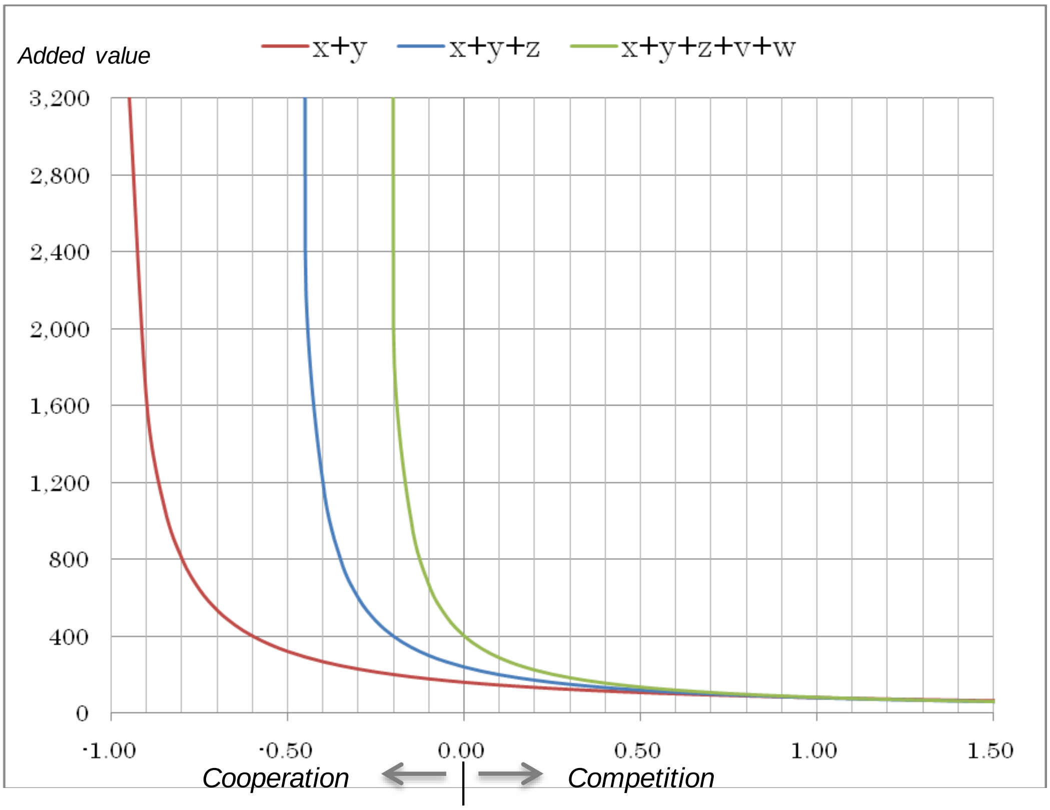
Figure 24-2



r1,r2=0.5, K1,K2=80, a,b=-0.5

Figure 25 shows how the competition coefficient decreasing affects the number of individuals when the number of species working together increases from two, three and five. It makes it clear that slowly increasing the negative competition coefficient results in the number of individuals increasing exponentially.

Figure 25



r = 0.5 each, K = 80 each, a,b = ±0.5 each

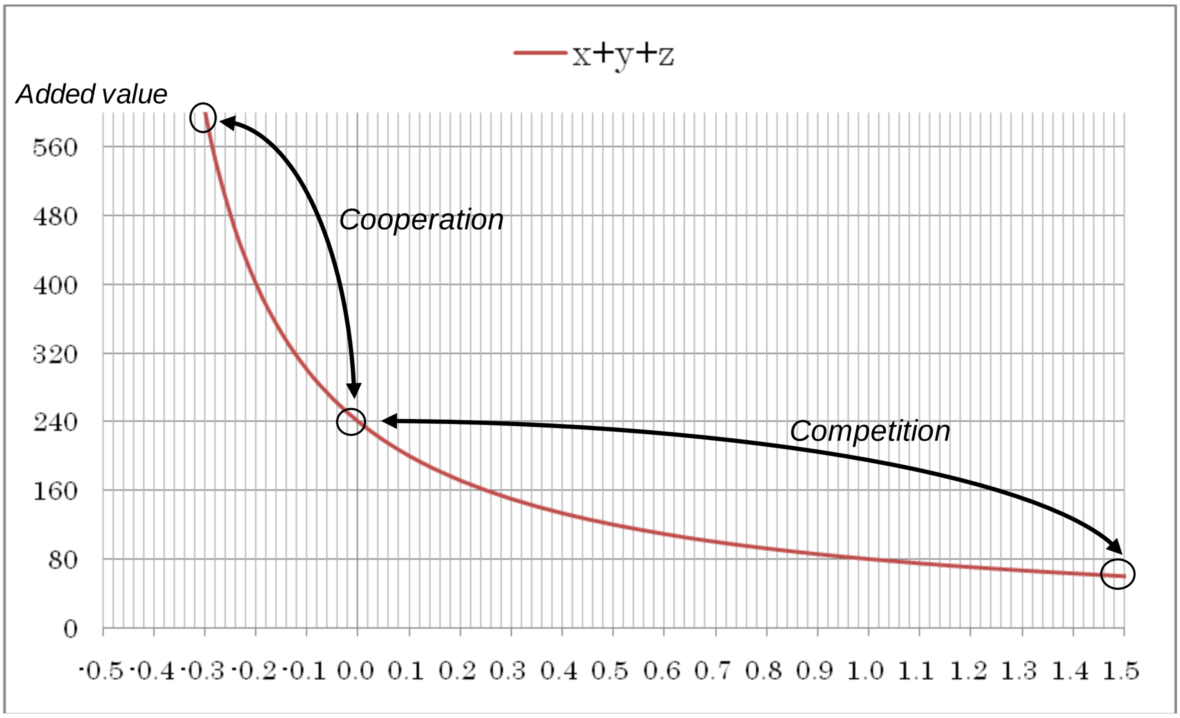
From now on, let's call the model with a positive competition coefficient the *"Competition Model"* and the model with a negative competition coefficient the *"Cooperation Model".* Let's take a look at what happens when the number of individuals in the *cooperation model* begin to increase.

＜Cooperation with companies in the same industry＞

Let's pretend there are three small manufacturers located close to each other that have similar products. In the *"Competition Model"*, the three companies would do things that limit each other's growth, for example one company might lower their prices. However, if the other two companies then follow and lower their prices, the positive effect for the first company doesn't last long, and eventually none of them make an ample profit. On the other hand, in the *"Cooperation Model"*, the three companies might cooperate to buy expensive manufacturing equipment. If they cooperate when buying and using it, the initial and fixed costs for each company are reduced by one third, and the productivity and quality provided by each company increase – resulting in the added value for the whole companies.

Let's think of the three companies as company ***x, y*** and ***z***. The growth of the added value of each company is shown in *Figure 26*. When the competition coefficient is *zero*, and the companies are completely independent, each company's added value is equal to the total of the carrying capacity (*240=80x3*). However, when the companies start acting in a way that limits their competitors growth, the competition coefficient moves to the right and becomes positive, meaning that the added value of each company ends up being limited. When the competition coefficient reaches *1.0*, the total added value of the three companies is the same as the carrying capacity of one company. In other words, it is reduced by one third. On the other hand, if they cooperate with each other, the competition coefficient moves to the left and becomes negative, meaning that the added value of the three companies instantly increases. For example, the total added value when the competition coefficient is *-0.20* is *400*, which is *1.7* times higher than it was when the companies were operating separately *(240)*. This proves that moving from competition to cooperation is a paradigm shift that has a significant impact on the fate of a company.

Figure 26



r=0.5, K=80 Vertical axis is the total added value of the three companies and the horizontal axis is the competition/cooperation coefficient.

＜Making Clusters＞

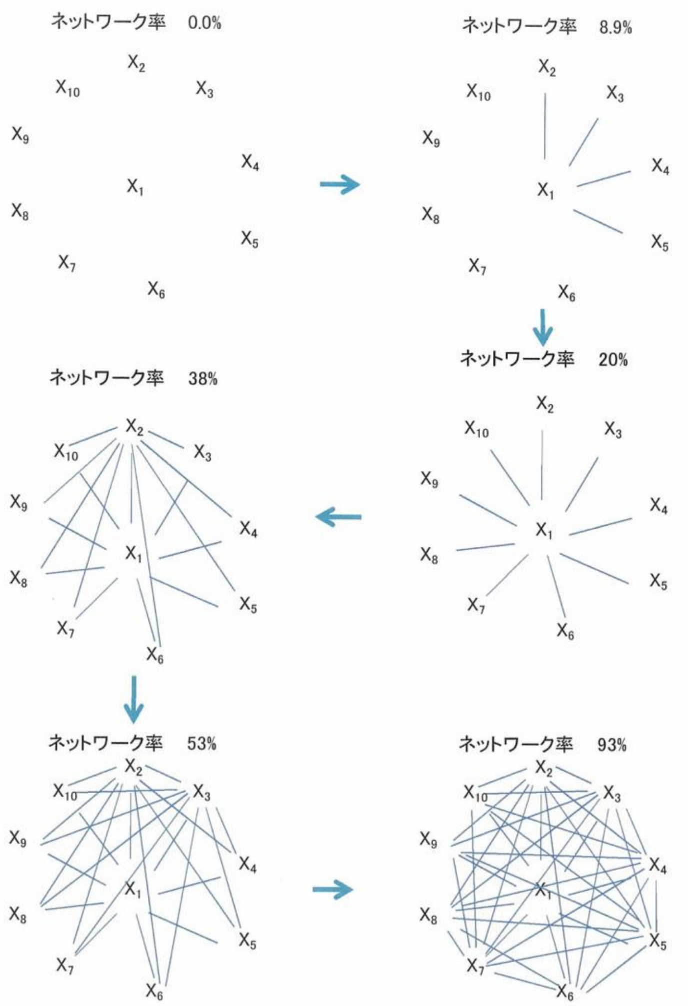
If we interpret the *cooperation model* as a *"network formation model"*, it becomes easier to explain how clusters and its added value develop. Let's call the individuals that make up the network *"agents"* and calculate how the added value changes when *agents* hold hands and create a cluster. The preconditions are outlined below.

Number of agents: 10
Growth rate (r) = 0.7 each
Carrying capacity (K) = 100 each
Cooperation coefficient: -0.1 each

Note) The differential equation used for the calculation is the same as the one used in Chapter 3 for the entire industry future prediction (3.2).

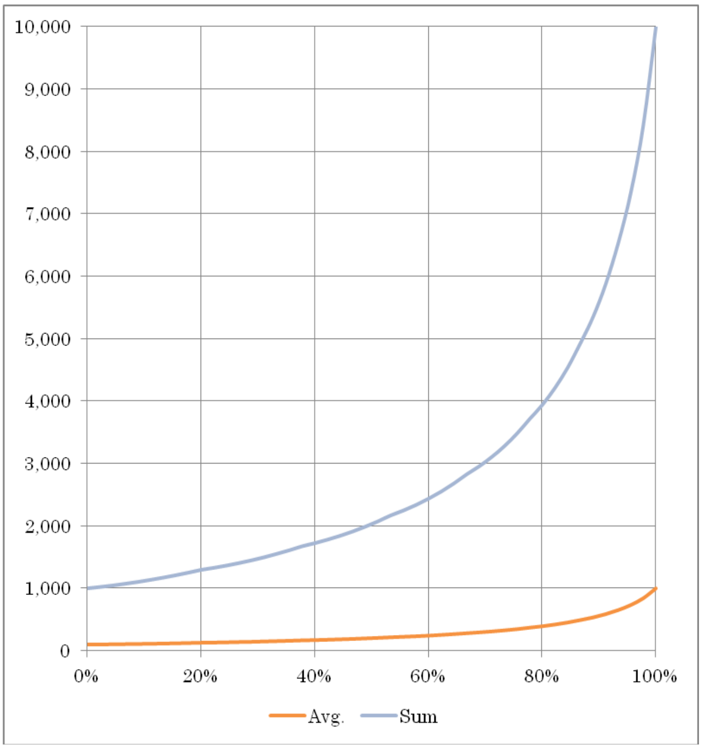
What happens when agents hold hands (*Figure 27*)

Figure 27



When the agents do not hold hands and are not interconnected in any way, the total added value never exceeds the total carrying capacity, which is *1,000 (100 x 10 agents)* (Orange line in *Figure 28*). When each agent begins cooperating with others to create a network, the added value begins to exceed the total carrying capacity and increase further. It is clear from the graph that as the percentage of agents holding hands increases, the added value increases exponentially. (Blue line in *Figure 28*)

Figure 28



Horizontal axis is percentage of agents holding hands, vertical axis is added value.

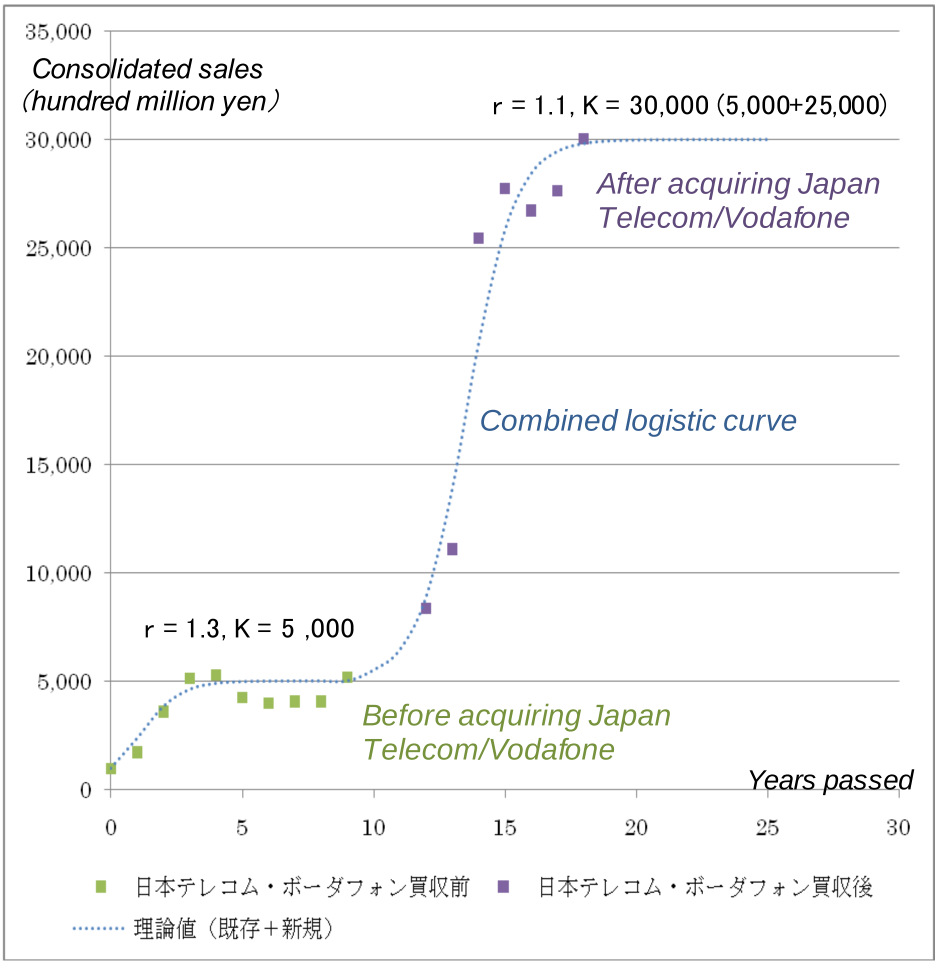
＜Growing through M&A＞

M&A can be thought of as a strategy to absorb competitors and turn competition into cooperation. From *Figure 27*, we can see that in the area that represents a negative competition coefficient, the more agents, the faster the speed of growth becomes. This is similar to how *M&A* can radically increase the size of a company in a short period of time. *Figure 29* represents the changes in consolidated sales and operating profits of *SoftBank Japan (former SoftBank Group Corp.)*. The company began in 1981 as a business distributing computer software packages. The founder, *Masayoshi Son*, continued to focus on higher goals and kept increasing their carrying capacity *K* and move into bigger industries to spur the company's growth. Their sales reached *100 billion yen 19* years after founding, *500 billion yen* after *16* years, *1 trillion yen* after *24* years and *3 trillion yen* after *29* years. (Operating profits from 2010 were the third highest in Japan) *Figure 29* shows this growth represented with a logistic growth curve, which was explained in *Chapter 1*. When the company was experiencing a plateau at five hundred billion yen, they acquired *Vodafone Japan*, allowing them to leave the plateau and quickly grow further. From the graph, it is clear that they are currently in their second plateau period (three trillion yen) .

SoftBank's M&A strategy can be seen as an application of the *cooperation model,* in which they acquired companies in the same industry to turn competition into cooperation. In 2010, *Masayoshi Son* announced his *30 year vision* for the company: *"I want to strategically break up the synergy groups and decentralize their management, creating a web type organization where each organization is independent, but works together. With moderate investment tie ups of 20 to 40%, I want to create a group that shares the same ambition and resolution. I want to expand this partner network strategy to include five thousand companies over the next 30 years."* Have another look at *Figure 29*. For as long as a company is operating with a positive competition coefficient, they cannot expect to grow significantly due to competition within their industry, but as soon as their competition coefficient becomes negative, they can experience fast growth.

Figure 29

Logistic curve analysis of Softbank consolidated sales, vertical axis is consolidated sales (hundred million yen), horizontal axis is years passed (1994=0)



<Chapter 5>

Expanding the model - fluctuations

According to information from the *Teikoku Databank* for corporate bankruptcy from 2000 to 2011, on average 10,000 companies went bankrupt each year. The Ministry of Internal Affairs reports that Japan is home to around 4 million companies, which makes the bankruptcy rate in Japan 0.25%. There are different opinions about whether this number is high or low, but after reading in *Chapter 2* how the environmental capacity and competition coefficient create patterns where certain companies are driven to bankruptcy by others, this number might be much lower than you were expecting. This number means that in the real world of business, 99.75% of businesses survive, and the process of selection and extinction outlined in this paper isn't occurring that much in the real business world. However, business failure does not always mean bankruptcy, it could also mean that a business leaves a certain market. In this case, the denominator would be much larger as it would represent the number of business projects within each company. But even in this instance, the ratio would likely be similar. In other words, in the real world of business, it is far more common for businesses to coexist while competing with each other. This chapter will examine why this is through mathematical principles.

Before we start looking at theories, let's take another look at the growth of the company that was examined in *Chapter 1*. Company sales revenue are usually made up of the combined sales revenue of a number of customer accounts. Sales from each customer account do not necessarily increase each year, they usually continue in a pattern of increasing and decreasing. The sales of the entire company are made up of all of these customer sales. *Figure 30* represents the fluctuation in sales by customer of the IBC rental company referenced in *Chapter 1*. From the graph, it is clear that the sales revenue from each customer account are constantly fluctuating. *Figure 31* shows the overall sales results from when all the customer account sales are added together. All of the lines from the first graph come together in one wave.

Figure 30
The fluctuations in sales by customer account in one company

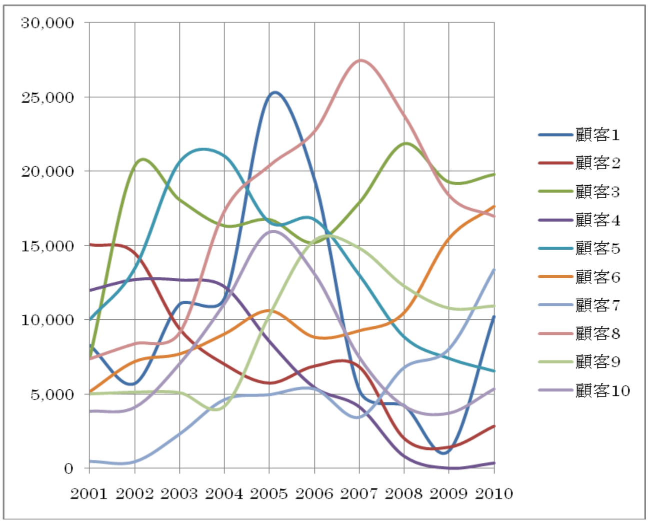
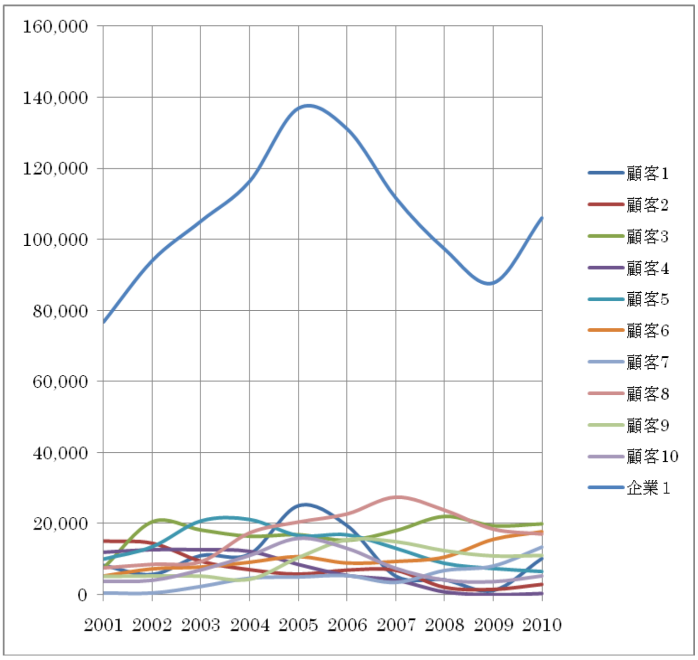


Figure 31

The fluctuation in total sales of a company is made up of the many fluctuations in sales for each customer account.



Each company in the industry has its own wave, and combining these waves allows us to see the fluctuations of the industry as a whole. These can then be overlaid on the waves of other industries in a country to make up the fluctuations of a country's economy. (*Figure 32*) In this way, looking closely at economic fluctuations, we can see how the fluctuations in customer account sales for one company can have an effect on the whole economy. And these fluctuations are the result of the competition coefficient of each company. In other words, this allows us to hypothesize that economic fluctuations occur due to the competition coefficient of each company constantly changing, creating a micro wave, which combines into a macro wave to create the economy.

Figure 32

Source: Japanese Cabinet (Economic index CI with 2005 at 100)

In the last section, we looked at what happens when the competition changes from positive to negative, but is important to note that these changes are not absolute. Like the saying “yesterday’s enemy is today’s ally”, it is common for companies that were once fierce competitors to end up working together. If a company makes a mistake with their strategy that benefits their competitor for a certain period of time, during that period the competition coefficient would change from positive to negative. There are also instances when changes in salespersons would result in new contracts being acquired etc. As shown with these examples, the competition coefficient in real life business is always changing. When the strongest players start to waver, the weaker players get a chance. This may be one of the reasons for the low bankruptcy rate shown earlier. Next let’s test this theory mathematically.

＜Competition Coefficient Fluctuations＞

Before we apply fluctuations to the competition coefficient and carrying capacity, let’s start by looking at the growth coefficient. The basic theory discussed in *Chapter 2* showed that the growth coefficient ***r*** does not usually affect the market share of the two companies, but what happens when fluctuations are applied? The differential equation *(2.1)* should be changed as shown in *(5.1).*

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1) \\ dy/dt&=r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

$$\downarrow$$

$$\left.\begin{aligned} dx/dt&=r_1\alpha(1+\cos w_1t)x(1-(x+by)/K_1) \\ dy/dt&=r_2\beta(1+\cos w_2t)y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (5.1)$$

Figure 33 shows the results of substituting the parameters from *Table 14* into the formula (5.1), setting an arbitrary initial point and graphing out the resulting growth locus. The point of coexistence is the same as before the fluctuations were reflected, *(x,y) = (60,50)*. The point of intersection, as discussed in Chapter 2, is as shown below, as it does not include the growth coefficient ***r***. Due to this, the point of coexistence does not change regardless of the fluctuation of ***r.***

$$\left.\begin{aligned} x &= (K_1-bK_2)/(1-ab) \\ y &= (K_2-aK_1)/(1-ab) \end{aligned}\right\} \quad (2.5)$$

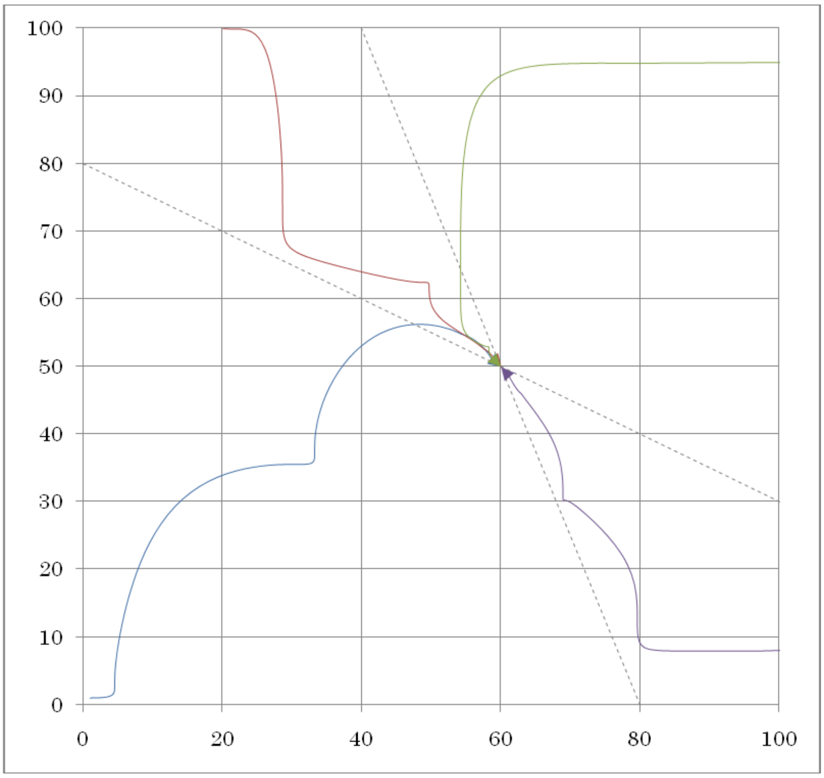
Table 14

r_1	0.5	a	0.5		
r_2	0.6	b	0.4	y の位相	3.0

K_1	80	α	0.5	w_1	0.5
K_2	80	β	0.5	w_2	0.4
⊿t	0.5				

Figure 33

Growth locus for when fluctuations are reflected in the growth coefficient. The point of coexistence is not affected.



＜Fluctuations in the competition coefficient＞

Next, let's see how competitive relationships are affected when the competition coefficient fluctuates. Let's start by examining the changes between two competing companies. The basic differential equation *(2.1)* will be edited as shown below in *(5.2)*.

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1) \\ dy/dt&=r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

↓

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+b(\cos w_2 t)y)/K_1) \\ dy/dt&=r_2y(1-(a(\cos w_1 t)x+y)/K_2) \end{aligned}\right\} \quad (5.2)$$

The fluctuation of the competition coefficient between positive and negative is represented with a *cosine wave*. The frequency of fluctuation (angular frequency) is represented as w_1 and w_2. t is time.

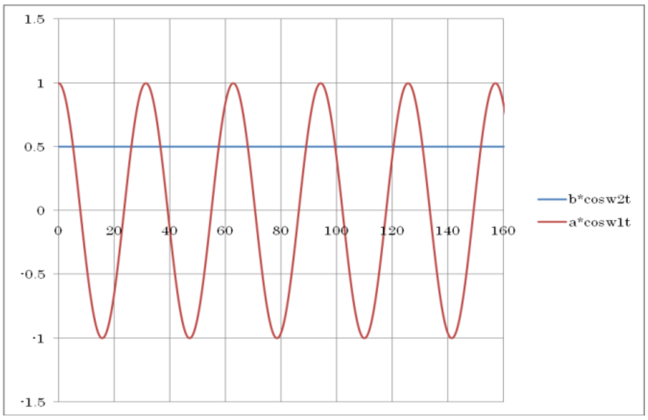
Let's set the coefficient as shown below to see how the relationship between the two companies is affected. First, let's just apply fluctuation to ***x***. The competition coefficient that represents how ***y*** is affected by ***x***, $a(\cos w_1 t)$ will have the w_1 changed with the values shown in *Table 15* to reflect fluctuation *(Figure 34)*.

Table 15

r_1	0.50	a	1.0000		
r_2	0.50	b	0.5000		
K_1	80	w_1	0.0000	→	0.2000
K_2	80	w_2	0.0000		

Figure 34

The fluctuations of the competition coefficient of x, $a(\cos w_1 t)$



When there is no movement, ***y*** loses out to ***x*** (*Figure 35-1*), but when ***x*** fluctuates, ***y*** is able to survive. (*Figure 35-2*) Additionally, if we look closely at the fluctuation, we can see an oval shape forming around the coordinates *(x,y) = (40,80).* It is clear that the value of ***y***, which was forced to leave the market before, is now relatively higher than ***x***.

Figure 35-1

y loses out to x
(r_1,r_2=0.5, K_1,K_2=80,
a=1.0, b=0.5, w_1=0.2, w_2=0)

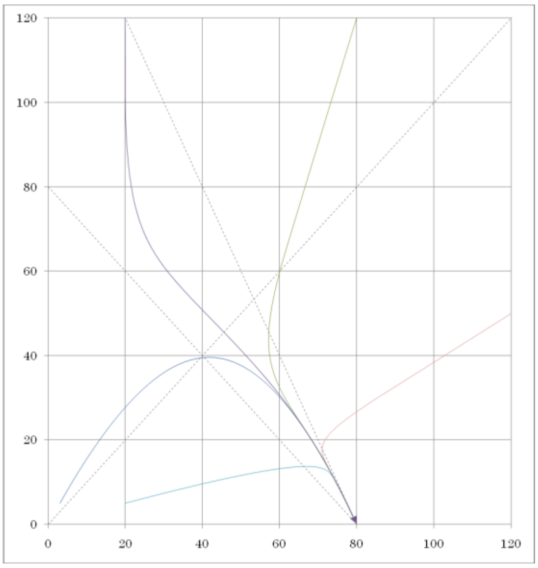
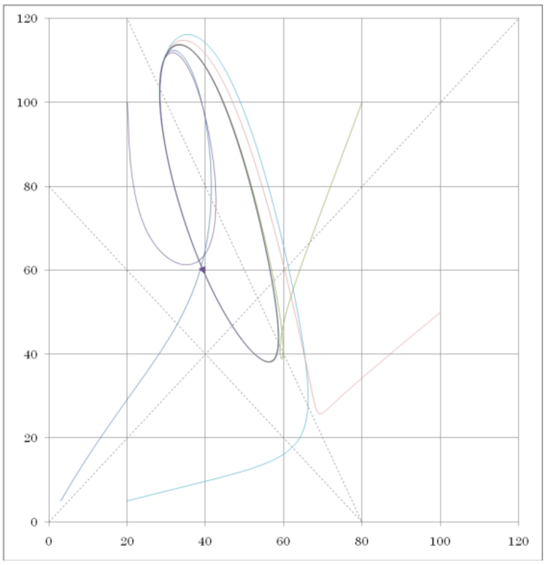


Figure 35-2

Adding fluctuation results in x & y coexisting
(r_1,r_2=0.5, K_1,K_2=80,
a=1.0, w_1=0,b=0.5,w_2=0)



Next, let's try adding fluctuation to both ***x*** and ***y***. Firstly take a look at the growth locus of ***x*** and ***y*** when the competition coefficient of the two companies is similar.

Table 16

r_1	0.50	a	1.1000
r_2	0.50	b	0.9000
K_1	90	w_1	1.0000
K_2	80	w_2	0.9900
⊿t	0.50		

Figure 36

Fluctuation of the competition coefficient (a=1.1, b=0.9, w_1=1.0, w_2=0.99)

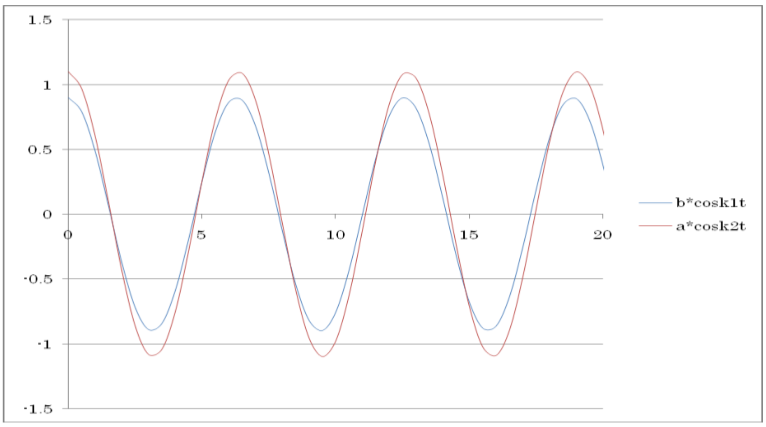


Figure 37-1 shows the situation when there is no fluctuation. Because the competition coefficient of ***x*** is high, ***y*** was forced out, but when we start to fluctuation on the competition coefficient, ***x*** and ***y*** are able to coexist (*Figure 37-2*). As time passes, the area in which the two lines move increases (*Figure 37-3, 37-4*), and they eventually (*t=650*) reach the same track (*limit cycle*) when ***x*** reaches between *35~130* and ***y*** reaches between *20~155* (*Figure 38-5*).

Figure37-1

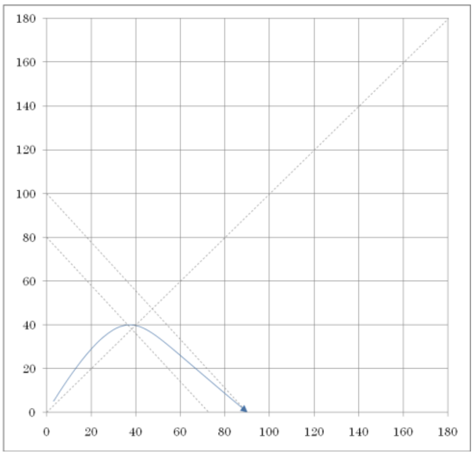


Figure 37-2

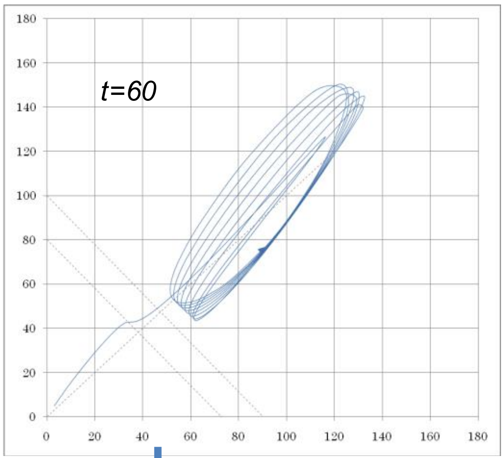


Figure37-3

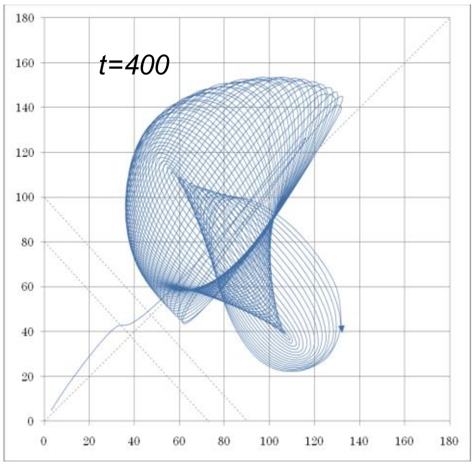


Figure 37-4

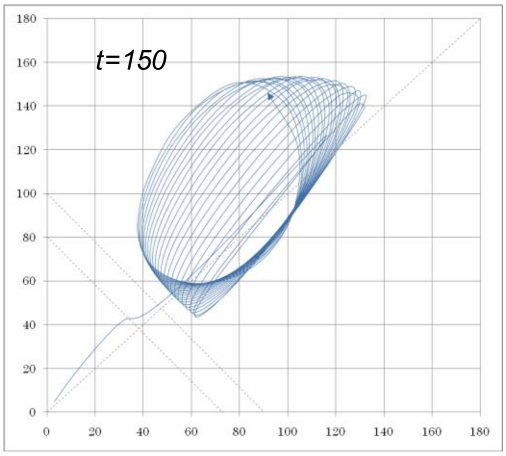
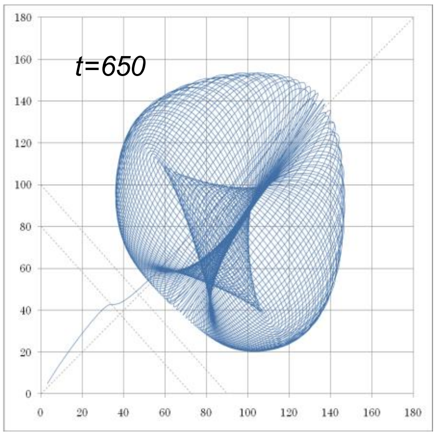


Figure 37-5



A limit cycle is illustrated with x having a scope of around 35 to 130 and y of around 20 to 155. (r_1,r_2=0.5, K_1=90,K_2=80, a=1.1, b=0.9, w_1=1.0, w_2=0.9)

Next, let's see what happens when we increase the numbers of competitors ***n*** from two to five, to represent five companies competing. The differential equation will be as below.

$$
\left.
\begin{array}{l}
dx/dt=r_1x(1-(x+b*\cos w_2t*y+c*\cos w_3t*z+d*\cos w_4t*v+e*\cos w_5t*w)/K_1) \\
dy/dt=r_2y(1-(a*\cos w_1t*x+y+c*\cos w_3t*z+d*\cos w_4t*v+e*\cos w_5t*w)/K_2) \\
dz/dt=r_3z(1-(a*\cos w_1t*x+b*\cos w_2t*y+z+d*\cos w_4t*v+e*\cos w_5t*w)/K_3) \\
dv/dt=r_4v(1-(a*\cos w_1t*x+b*\cos w_2t*y+c*\cos w_3*z+v+e*\cos w_5t*w)/K_4) \\
dw/dt=r_5w(1-(a*\cos w_1t*x+b*\cos w_2t*y+c*\cos w_3*z+d*\cos w_4t*v+e)/K_5)
\end{array}
\right\} \quad (5.3)
$$

Each parameter will be set as below for the time being.

Table 17

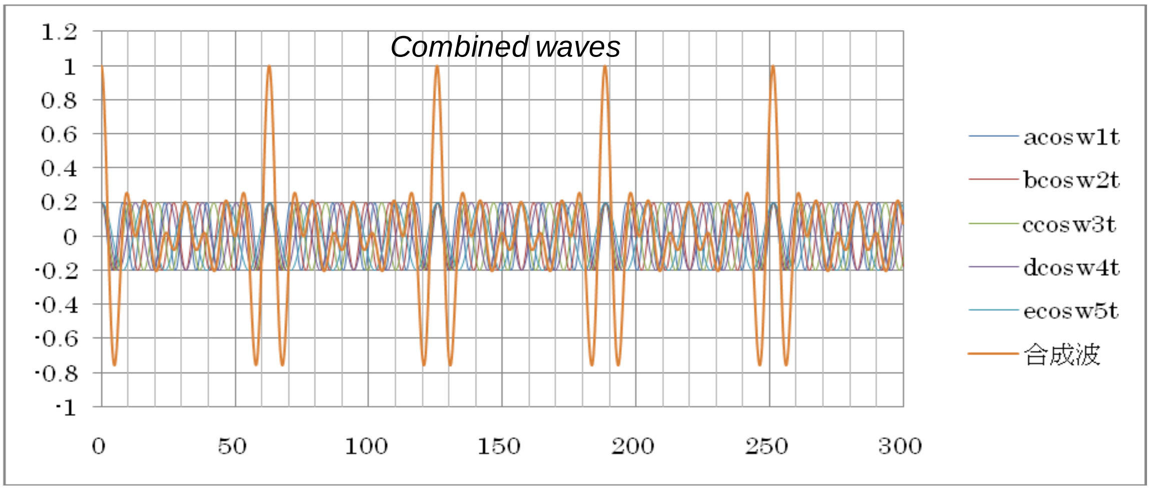
	Growth Coefficient	Carrying Capacity	Competition Coefficient			
	r	K	Amplitude		Angular frequency	
x	0.5	100	a	0.20	w_1	0.80
y	0.4	90	b	0.20	w_2	0.70
z	0.3	80	c	0.20	w_3	0.60
v	0.2	70	d	0.20	w_4	0.50
w	0.1	60	e	0.20	w_5	0.40
計		400				

Firstly, let's have a focus on the *competition coefficient* of *(5.4)* and replace it with the parameters shown in *Table 17*. See *Figure 38*. We can see that each of the five competition coefficients are frequently fluctuating. We can also see that each wave comes together to form a bigger wave that moves in one large cycle. If we use the angular frequency to find each company's cycle period and calculate the cycles during each company is synced, we can see that the cycles sync at $t = 62.83$ and create the biggest combined wave. They sync again as shown below in multiples of $t = 62.83$. (angular frequency $w = 2\pi f$. Here the ***f*** refers to the frequency. The period T is the reciprocal of the frequency f, $T=1/f$. Thereby, the period can be obtained by calculating the frequency from the angular frequency and then finding the reciprocal of that.)

	Period		*Cycle*		*Time synced*
x	*7.85*	*x*	*8*	=	*62.83*
y	*8.98*	*x*	*7*	=	*62.83*
z	*10.47*	*x*	*6*	=	*62.83*
v	*12.57*	*x*	*5*	=	*62.83*
w	*15.71*	*x*	*4*	=	*62.83*

Figure 38

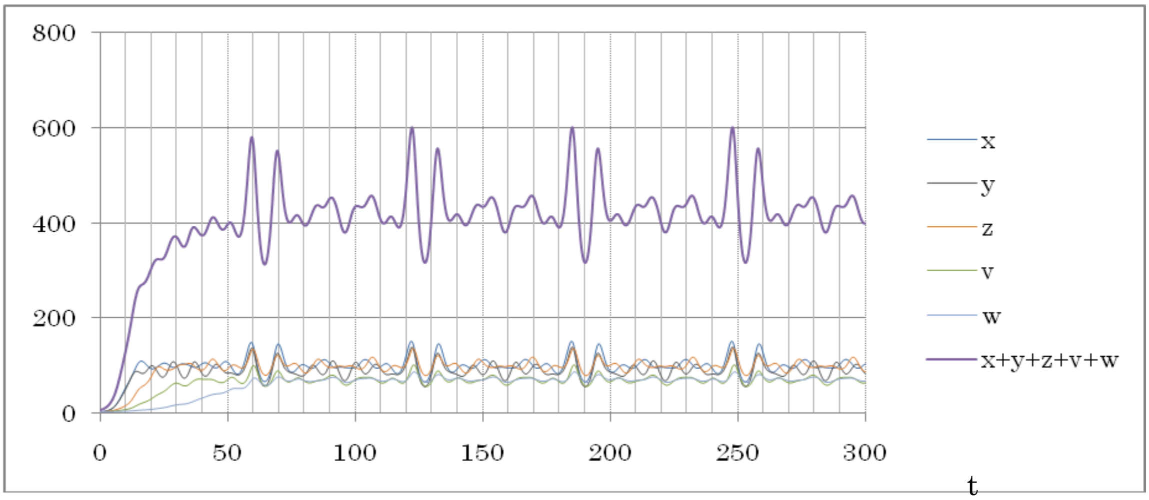
Change in competition coefficient fluctuation. The combined wave syncs in multiples of t=62.83



Next, let's substitute in the change in competition coefficient into *(5.3)* and see how the sales of the five companies change *(Figure 39)*. When the competition coefficient is a high positive number (meaning that each company limits the other's growth), the sales of all of the five companies starts to decrease. However, when the competition coefficient is negative (meaning that the companies are cooperating with each other), the sales of the companies start to increase. Therefore, it can be seen that the combined wave of the competition coefficient and the combined wave of the sales of each of the companies are almost symmetrical when compared on a time axis.

Figure 39

The overall sales of the five companies (x,y,z,v,w) fluctuates along with the competition coefficient.



Next, lets increase the amplitude and decrease the fluctuation rate of the *competition coefficient*.

Table 18

	Growth Coefficient	*Carryingl Capacity*	*Competition Coefficient*			
	r	*K*	*Amplitude*		*Angular frequency*	
x	*0.5*	*100*	*a*	*0.85*	w_1	*0.30*
y	*0.4*	*90*	*b*	*0.75*	w_2	*0.15*
z	*0.3*	*80*	*c*	*0.90*	w_3	*0.10*
v	*0.2*	*70*	*d*	*0.80*	w_4	*0.20*
w	*0.1*	*60*	*e*	*0.70*	w_5	*0.15*
合計		*400*				

If we calculate the cycle and sync timing, we can see that this case is two times longer than the previous case.

	Period		*Cycle*		*Time synced*
x	*20.94*	*x*	*6*	=	*125.66*
y	*41.89*	*x*	*3*	=	*125.66*
z	*62.83*	*x*	*2*	=	*125.66*
v	*31.42*	*x*	*4*	=	*125.66*
w	*41.89*	*x*	*3*	=	*125.66*

When *t* = *125.66*, the cycles sync and the combined wave reaches its largest. It also syncs when it reaches the multiples outlined below. The highest amplitude of the competition coefficient wave is *4.0*, which is four times higher than the previous case *(Figure 39*). Additionally, the total sales of the five companies also fluctuates greatly along with the competition coefficient. The frequency is about two to three times more than the previous case *(Figure 40*).

Figure 40

Increasing the amplitude of competition coefficient fluctuations will increase the size of the combined wave. At its maximum value of 4.0, it is 4 times higher than in Figure 39.

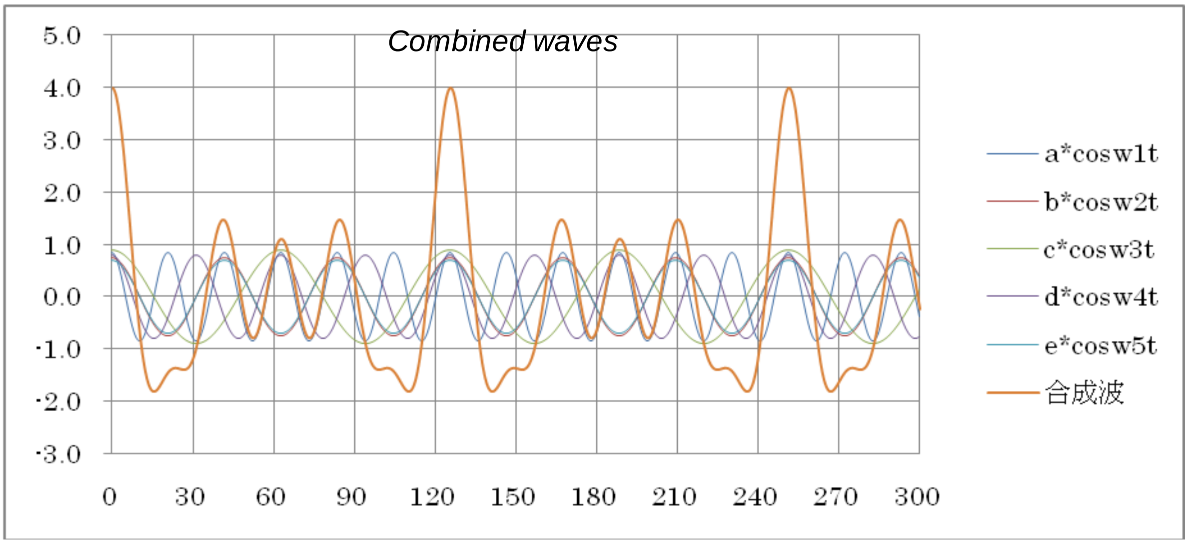
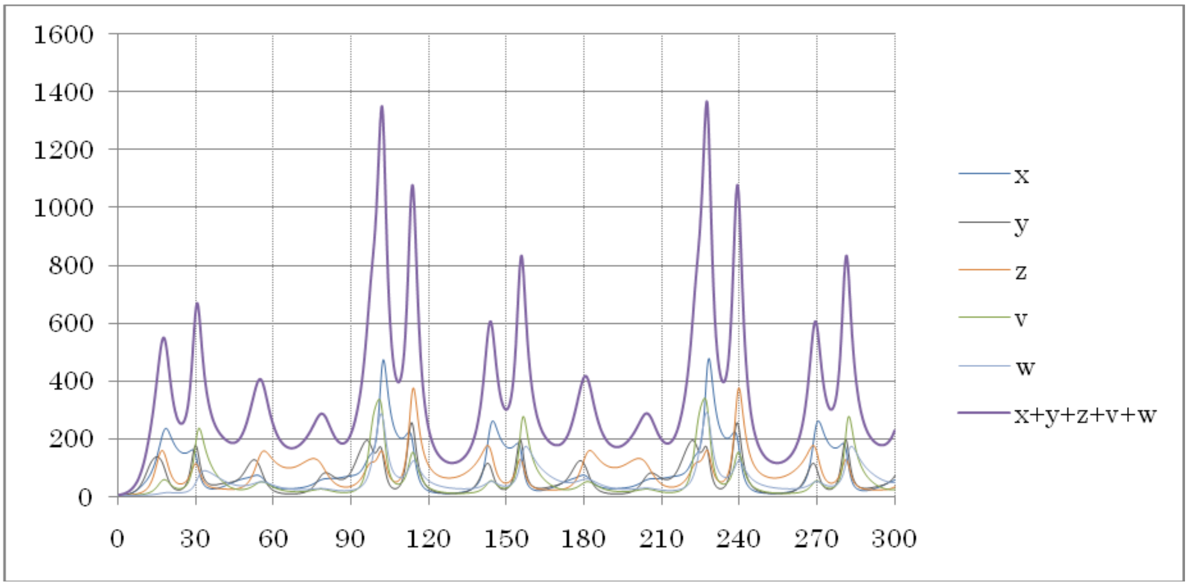


Figure 41

The total sales of the five companies (x,y,z,v,w) fluctuates greatly along with the competition coefficient. The scope of fluctuation is two to three times higher than in Figure 40.



<Fluctuations of the Carrying Capacity>

Next, let's take a look at fluctuations of the *carrying capacity*. Firstly, what happens when the carrying capacity fluctuates? To find out, let's return to basics with just one company. Change the differential equation for one species (company) *(1.3)* to *(5.4)*. Adding *1* to cosine is done in order to ensure that the carrying capacity does not become a negative number.

$$dx/dt = rx(1-x/K) \quad (1.3)$$

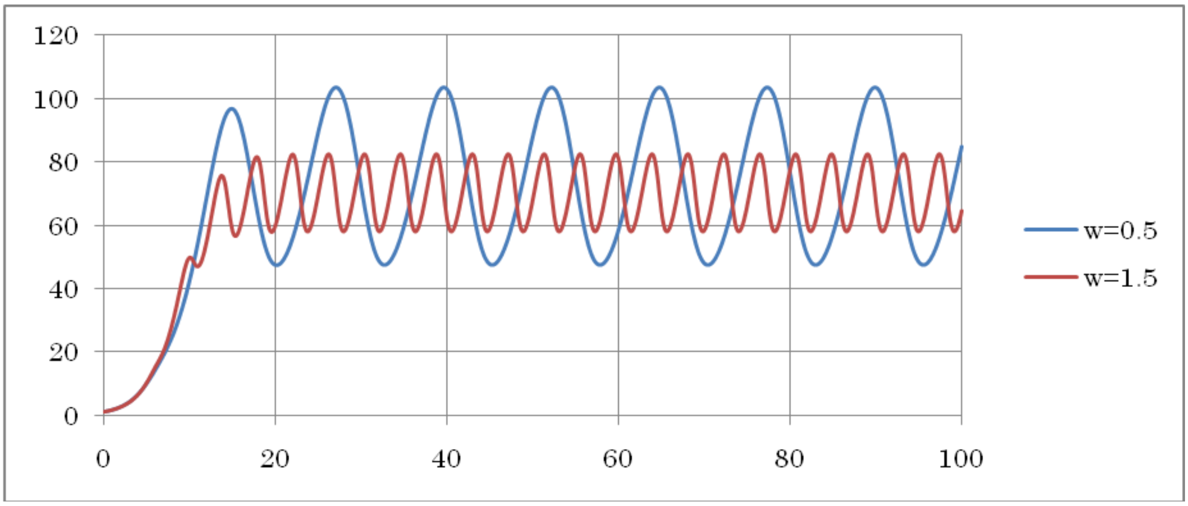
$$\downarrow$$

$$dx/dt=rx(1-x/K\ (1+\alpha\cos wt)) \quad (5.4)$$

If we set the coefficient as below and graph the logistic curve for *(5.4)*, we can easily see how the fluctuation of the *carrying capacity* is having an effect. (*Figure 42*). The values fluctuate up and down for the *carrying capacity K=80*. If we increase the angular frequency ***w***, we can see that the density of the waves increases (the changes become more frequent).

Figure 42

Fluctuations in carryingl capacity and individual numbers (sales)
r=0.5, K=80, Amplitude α=0.5, Angular frequency w=0.5 (blue line), w=1.5 (red line)



Now that we understand what happens when there is only one species (company), let's try applying the model to a situation with two. First, we will rewrite *(2.1)* like *(5.5).*

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1) \\ dy/dt&=r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

$$\downarrow$$

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1\ (1+\alpha\cos w_1t) \\ dy/dt&=r_2y(1-(ax+y)/K_2(1+\beta\cos w_2t) \end{aligned}\right\} \quad (5.5)$$

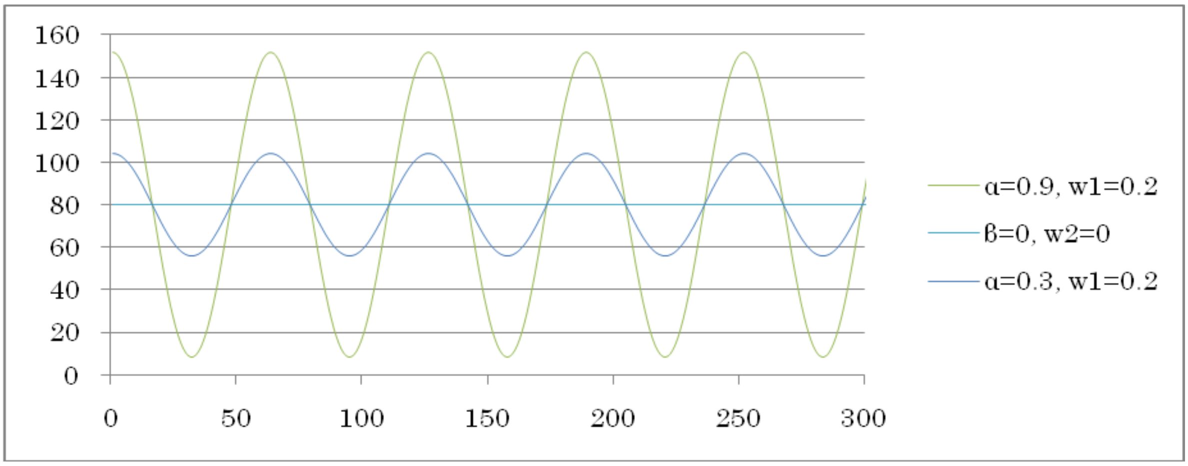
Firstly, if we set the basic conditions without fluctuations as shown in *Table 22*, ***x*** and ***y*** come together at the point of intersection *(57,46) (Figure 43).*

Table 19

r_1	0.5	a	0.6
r_2	0.5	b	0.5
K_1	80.0		
K_2	80.0		

Figure 43

*Fluctuations in **x**'s carrying capacity, K_1*



Let's reflect the below fluctuations on only the *carrying capacity* of ***x.***

Table 20

r_1	0.5	a	0.6		
r_2	0.5	b	0.5		
K_1	80.0	**α**	**0.300**	w_1	**0.200**
K_2	80.0	β	0.000	w_2	0.000

Now let's amplify the fluctuations.

Table 21

r_1	0.5	a	0.6		
r_2	0.5	b	0.5		
K_1	80.0	**α**	**0.900**	w_1	**0.200**
K_2	80.0	β	0.000	w_2	0.000

What are the results of this like? The below graph shows the changes in ***x*** and ***y*** when the conditions in *Tables 19 to 21* are applied.

Figure 44-1

*When neither **x** or **y** fluctuate, the convergence is (57,46)*
$r_1, r_2=0.5$, $a=0.6$, $b=0.5$, $K_1, K_2=80$

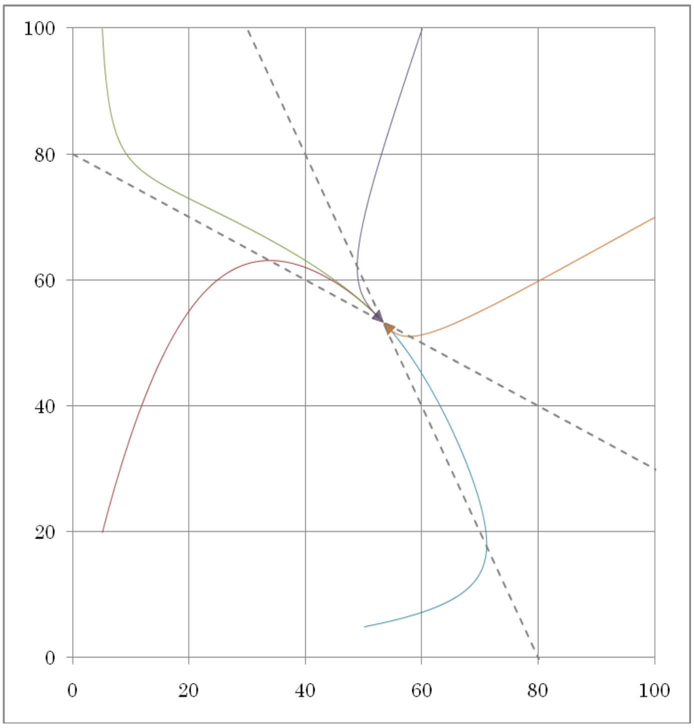


Figure 44-2

*Applying fluctuations to **x**'s carrying capacity, **K_1** creates an ellipse shape centered around the abscissa of convergence. r_1, r_2=0.5, a=0.6, b=0.5, K_1=80(1+0.3cos0.2t),,K_2=80*

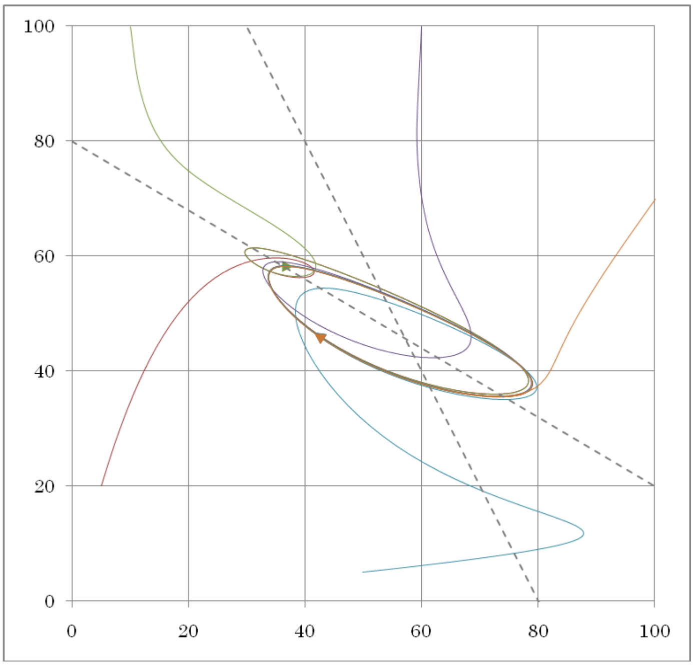
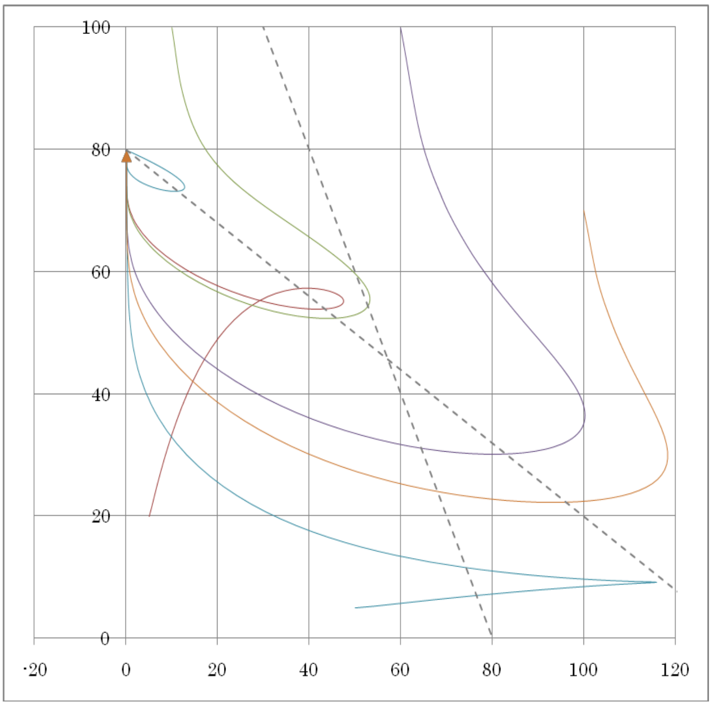


Figure 44-3

*Applying larger fluctuations to **x**'s carrying capacity **K_1** results in **x** losing out to **y.***
r_1, r_2=0.5, a=0.6, b=0.5, K_1=80(1+0.3cos0.2t),,K_2=80



The fluctuations of the Carrying capacity can be seen as changes in the management goals of the company. Based on the results of the calculations above, it is clear that the more that the goals change, the higher the risk that the business will not be able to survive. In Chapter 3, there was a warning for management that is focused on maintaining their company's current position. These results also prove that managers should have consistently high goals for their company.

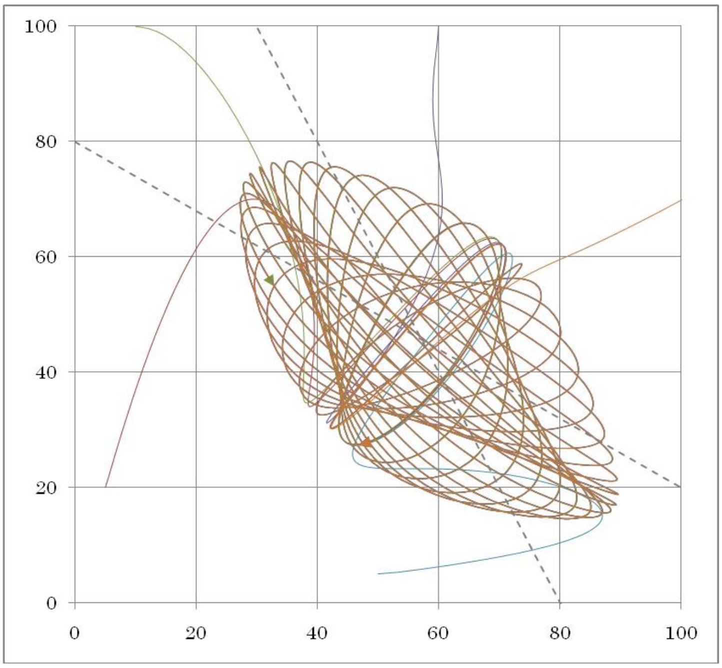
Next, lets apply the same level of fluctuation as ***x*** from the last example to ***y***'s *carrying capacity,* $\boldsymbol{K_2}$, as shown in *Table 22*. *Figure 45* shows the results of this. When comparing with *Figure 44-2*'s ellipse shape, we can see that the area covered by the graph is much larger, despite the fact that the coefficient was only slightly changed.

Table 22

r_1	0.5	a	0.6		
r_2	0.5	b	0.5		
K_1	80.0	α	0.300	w_1	0.200
K_2	80.0	β	0.320	w_2	0.210

Figure 45

*When the carryingl capacities of both **x** and **y** fluctuate around the same value. The scope of the lines increase compared to when only **x**'s carrying capacity fluctuated.*



Finally, let's add fluctuations to both the competition coefficient and the environmental capacity. The differential equation we'll use will be *(5.8),* which was created with both *(5.5)* and *(5.6)*.

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1) \\ dy/dt&=r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

↓

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+b(\cos w_2t)y)/K_1) \\ dy/dt&=r_2y(1-(a(\cos w_1t)x+y)/K_2) \end{aligned}\right\} \quad (5.5)$$

↓

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+by)/K_1\ (1+\alpha\cos w_1t) \\ dy/dt&=r_2y(1-(ax+y)/K_2(1+\beta\cos w_2t) \end{aligned}\right\} \quad (5.6)$$

↓

$$\left.\begin{aligned} dx/dt&=r_1x(1-(x+b(\cos k_1t)y)/K_1\ (1+\alpha\cos w_1t) \\ dy/dt&=r_2y(1-(a(\cos k_2t)x+y)/K_2(1+\beta\cos w_2t) \end{aligned}\right\} \quad (5.8)$$

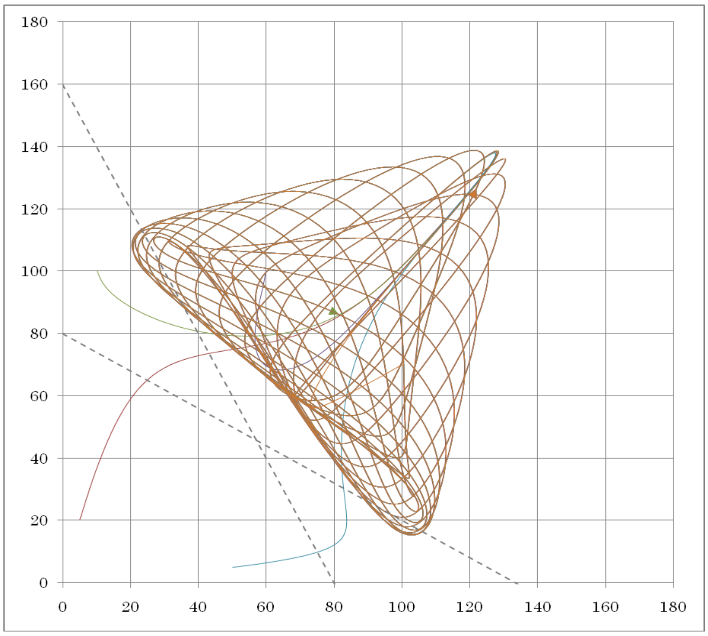
Each coefficient is as shown in *Table 23.*

Table 23

r_1	0.5	a	0.6	k_1	0.200
r_2	0.5	b	0.5	k_2	0.210
K_1	80.0	α	0.200	w_1	0.200
K_2	80.0	β	0.210	w_2	0.210

Figure 46

When the competition coefficient and carrying capacity of x and y fluctuate. The scope of the lines increases even more.



What do these results show? Let's go back to the start to find out. According to the basic theory from Chapter 2, the combination of certain values for the competition coefficient and carrying capacity result in either ***x*** or ***y*** losing out, or in both coexisting together. When the coefficients are fixed, there is only one point of coexistence, and the values will end up at that point regardless of the initial value. After that point, both ***x*** and ***y*** will keep staying at the coexistence point, regardless of how much time passes. However, in the real world of business, the coefficient values are never fixed. In the real world, coexistence is not static, and changes are always taking place. This is what allows each company chances and creates risks.

- The end of Chapter 5 -